647510 **BSc4MatC0401** Seat No :________

B.Sc. Semester - 4 (*CBCS*) Examination

April/May -2022 [OLD COURSE]

MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any one of the following questions. **[07]**

(1) Prove in usual notations that $(R^3, +, \cdot)$ is a vector space over R.

(2) Let $A = \{(2, 1, 0), (-1, 0, 1), (0, 1, 2)\}$. Check whether $(0, -1, 1), (1, 0, -1) \in Sp(A)$ or not.

Q.1 (B) Answer any one of the following questions. **[04]**

(1) State and prove necessary and sufficient conditions for a non-empty subset W of a vector space V to be a subspace of V.

(2) Prove or disprove: union of two subspaces of a vector space V is also a subspace of V.

Q.1 (C) Answer any one of the following questions. **[03]**

(1) Let B be a linearly dependent set of vectors and A be a finite subset of V such that $B \subseteq A$. Show that A is also linearly dependent.

(2) Check whether the set $\{2, 1 + x, x^2\}$ is linearly independent or not in $P_2(R)$.

Q. 2 (A) Answer any one of the following questions. **[07]**

(1) Let W_1 and W_2 be two subspaces of a finite dimensional vector space V. Show that $dim(W_1 + W_2) = dim(W_1) + dim(W_2) - dim(W_1 \cap W_2)$.

(2) Prove that every linearly independent set of vectors can be extended to a basis.

Q.2 (B) Answer any one of the following questions. **[04]**

(1) Check whether the set $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ is a basis of R^3 or not.

(2) Extend the set $A = \{(1, 1, 1, 1), (1, 2, 1, 2)\}$ to form a basis of R^4.

Q.2 (C) Answer any one of the following questions. **[03]**

(1) Show that numbers of elements in any two distinct basis of a finite dimensional vector space are equal.

(2) Find co-ordinates of vector $(1, 1, 0)$ with respect to the basis $\{(2, 2, 3), (2, 1, 3), (1, 0, 1)\}$ of R^3.

Q.3 (A) Answer any one of the following questions. **[07]**

(1) State and prove rank nullity theorem.

(2) Find N_T, R_T, $n(T)$ and $r(T)$ for the linear transformation $T : R^3 \rightarrow R^2$ defined as $T(x, y, z) = (x - y + z, x + y - z)$; $\forall (x, y, z) \in R^3$.

Q.3 (B) Answer any one of the following questions. **[04]**

(1) State and prove necessary and sufficient conditions for a linear transformation $T : U \rightarrow V$ to be one-one.

(2) Let $T: U \rightarrow V$ be a linear transformation. If $U= sp\{u_1, u_2, \ldots, u_n\}$, then show that $R_T = sp\{T(u_1), T(u_2), \ldots, T(u_n)\}$.

Q.3 (C) Answer any one of the following questions. **[03]**

(1) Let $T:U \rightarrow V$ be a linear transformation. In usual notations prove that $Ker(T)$ is a subspace of U.

(2) Find a linear transformation $T : R^3 \rightarrow R^3$ such that $T(e_1) = e_1 + e_2$; $T(e_2) = e_2 + e_3$ and $T(e_3) = e_1 + e_2 + e_3$ where $\{e_1, e_2, e_3\}$ is the standard basis of R^3.

Q.4 (A) Answer any one of the following questions. **[07]**

(1) Let $T : R^3 \to R^4$ defined as $T(x, y, z) = (x + y, x- y + 2z, z, 0); \forall (x, y, z) \in R^3$. Let $B_1 = \{(1, 1, 0), (1, -1, 0), (0, 2, 1)\}$ and B_2 be the standard basis of R^4. Find $[T ; B_1, B_2]$.

(2) Let $T : R^2 \to R^2$ defined as $T(x, y) = (3y, 2x - y); \forall (x, y) \in R^2$. Find eigen values and eigen vectors of T.

Q.4 (B) Answer any one of the following questions. **[04]**

(1) Let $T : V \to V$ be a linear transformation and B be a basis of V. Show that T is singular iff $det[T; B] = 0$.

(2) In usual notations show that L(U, U) is an algebra.

Q.4 (C) Answer any one of the following questions. **[03]**

(1) Define Linear functional and Dual of a vector space.

(2) Let $T : V \to V$ be a linear transformation and B be a basis of V. Show that λ is an eigen value of T if $det([T- \lambda I_n , B]) = 0$.

Q.5 (A) Answer any one of the following questions. **[07]**

(1) Find radius of curvature of the curve given by $x = a (t + sint), y = a(1 – cos t); t \in [0, 2\pi]$.

(2) Discuss nature of double points of the curve $x^3 + y^3 - 3x^2 -3xy + 3x +3y – 1 =0$.

Q.5 (B) Answer any one of the following questions. **[04]**

(1) Find all asymptotes of the parabola $y^2 = 4x$.

(2) Find radius of curvature at (2, 0) of the circle $x^2 + y^2 = 4$.

Q.5 (C) Answer any one of the following questions. **[03]**

(1) Show that the origin is a point of inflexion for the curve $y = x^5$.

(2) Find asymptotes parallel to co-ordinate axis of the curve $4 x^2 -9 y^2 = x^2 y^2$.

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